Offline impairment-aware RWA and regenerator placement in translucent optical networks

B. Garcia-Manrubia, P. Pavon-Marino, R. Aparicio-Pardo, M. Klinkowski and D. Careglio

Abstract-In translucent optical networks the physical layer impairments degrading the optical signal are considered in the network planning. In this paper we investigate the offline problem of Routing and Wavelength Assignment (RWA) and Regenerator Placement (RP) in translucent networks, minimizing the lightpath blocking and regenerator equipment cost. We address two variants of the problem, which correspond to two different types of Quality of Transmission (QoT) estimators, called Linear and Non-Linear. In a Non-Linear QoT, non-linear impairments like crosstalk or cross-phase modulation which account for the interferences from neighboring lightpaths in the network are explicitly computed. Then, the QoT estimated for a lightpath depends on the routes of other lightpaths in the network. In the Linear QoT the effects of the non-linear impairments are over-estimated and accumulated to the rest of the impairments in the OoT calculation. As a result, the OoT estimation of a lightpath solely depends on its route.

For the linear case, we formulate an optimal ILP model of the problem, to the best of the authors' knowledge, for the first time in the literature. Its simplicity allows us to test it for small and medium size networks. Also, we propose two heuristic methods, namely Lightpath Segmentation and 3-Step, and a tight lower bound for the regenerator equipment cost. For the non-linear QoT case, we propose a new heuristic called Iterative Regenerator Placement (IRP). Both the IRP and 3-Step algorithms are designed to guarantee that no lightpath blocking is produced by signal degradation. This is a relevant difference with respect to previous proposals. The performance and the scalability of our proposals are then investigated by carrying out extensive tests. Results reveal that the solutions obtained by the heuristic algorithms are optimal or close-to-optimal, and outperform the previous proposals in the literature.

Index Terms—Impairment Aware Network Planning, Regenerator Placement, Translucent Optical Networks.

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I. INTRODUCTION

THE popularization of bandwidth-hungry applications and services is enforcing the deployment of optical WDM networks with a clear trend towards increasing the capacity and lowering the network cost (both CAPEX and OPEX). On the one hand, this trend can be translated to higher, 40/100 Gbps line rates and denser WDM transmission systems with 80 to 160 wavelengths per fiber. On the other hand, aiming at reducing the cost, recent advances in optical technologies are fostering an evolution from traditional opaque to transparent optical network architectures [1].

In an opaque network, each node is equipped with Optical Electrical Optical (OEO) interfaces meaning that the optical signal carrying traffic terminates at each node to undergo an OEO conversion and an electronic processing. This approach simplifies the network design and control since there is a full independence between the network and the physical layer. On the contrary, it requires a large amount of OEO devices greatly increasing the network cost and the energy consumption.

In transparent optical networks, the optical signal originated at the source nodes reaches its destination bypassing optically the intermediate nodes. This approach reduces considerably the cost since neither OEO conversions nor electronic processing is required at each node along a lightpath. However, it implies that the physical layer must support end-to-end communication. Unluckily, the transmission reach of optical signals is limited due to the accumulation of physical layer impairments which cause transmitted data to not be received correctly, e.g. the Bit Error Rate (BER) is higher than an acceptable threshold.

For that very reason, translucent (or semi-transparent) optical networks are emerging as a promising solution for bridging the gap between opaque and transparent networks. Indeed, translucent networks combine features of both opaque and transparent networks strategically placing electrical regeneration (i.e. by means of OEO conversion) only at selected points in the network [2]. This approach eliminates much of the required electronic processing and allows a signal to remain in the optical domain for much of its path. Moreover, an electrical regenerator enables the possibility of wavelength conversion which may help to decrease the number of rejected lightpaths compared to the transparent case [3].

In both transparent and translucent optical networks, the network and the physical layer cannot be decoupled (as in opaque networks) and a cross-layer design is necessary. The traditional problem of routing and wavelength assignment (RWA) must take into account the signal impairments in the lightpath computation process in the so called impairment aware RWA (IA-RWA). Two cases can be faced. During the planning phase, the traffic demand is already known at least partially; therefore the decisions can be taken offline using static IA-RWA algorithms. The other case, whereby traffic demands are assumed to arrive in a dynamic fashion, is referred to as the online or dynamic IA-RWA problem. A comprehensive literature review of proposed static and dynamic IA-RWA algorithms can be found in [4].

Besides, in the translucent optical networks, there are the additional problems of regenerator placement and allocation. In the planning phase, the regenerator placement consists of selecting which nodes of the network have regeneration capabilities and how many signals can be regenerated at these nodes. In contrast, in the operation phase, the regenerator allocation tries to determine how the already placed regenerators are used in a dynamic scenario.

In all these issues, a Quality of Transmission (QoT) estimator accounting for the accumulation of the physical layer impairments along the path and thus determining the signal QoT is needed. These impairments include chromatic and polarization-mode dispersion, optical-fiber nonlinearities, noise accumulated due to amplified spontaneous emission (ASE); crosstalk; etc. In literature, there are three main QoT estimators [5] based on the numerical calculation of the Optical Signal-Noise Ratio (OSNR) [6], applying analytical formulas [7] or interpolating numerical and laboratory measurements [8], [9] to compute the Q factor value. Note that the Q factor of a lightpath is in direct relation to its signal Bit Error Rate (BER) performance [10]. For example, to evaluate the feasibility of a lightpath, its QoT in terms of OSNR must be higher than (typically) 19 dB for a 10G system; it must be lower than 10⁻⁹ in the case of BER; or it must be higher than 15.5 dB in case of Q factor.

In this paper we focus on translucent optical networks and we address the offline problem of network planning where, given a network topology and an estimation of the traffic demands, both the static IA-RWA and the regenerator placement problems are solved jointly. We denote this problem as IA-RWA-RP (IA-RWA-Regenerator Placement). As a QoT estimator, we use two different models which we denote as Linear and Non-Linear QoT. In both cases, a lightpath is accepted if its QoT is higher than a given threshold. The two models are representative of two different strategies of accounting for the so-called non-linear impairments. In this context, non-linear impairments refer to signal degradations in a lightpath caused by neighboring lightpaths (e.g. crosstalk or cross-phase modulation). In its turn, linear impairments refer to those which depend only on the lightpath route and wavelength (e.g. ASE). In a Non-Linear QoT estimator, non-linear impairments are explicitly computed. Then, the QoT obtained for a lightpath is dependent on the routes of other lightpaths in the network. In the Linear QoT the effects of the non-linear impairments are over-estimated and accumulated to the linear ones. As a result, the QoT estimation of a lightpath solely depends on its route. In this paper, we use the extended Q-Personick model defined in [9] as a representative of Linear QoT estimator. For the Non-Linear QoT case, we implement a *Q* factor based on analytical models [11]-[16]. We denote it as Q-NL factor.

The two QoT models trigger different algorithmic approaches to the planning problem. For this reason, we denote the associated problem variants as Linear IA-RWA-RP and Non-Linear IA-RWA-RP respectively. In both cases, our target is to find the network planning which minimizes both (i) the lightpath blocking caused by the optical signal degradation and wavelength conversion requirements, and (ii) the cost of the regeneration equipment. We denote the lightpath blocking directly caused by the signal degradation wavelength conversion requirements as signal regeneration blocking. Note that in the offline planning of translucent networks there is other source of lightpath blocking: the one caused by the limited number of wavelengths of the links. We name this second type of blocking as network capacity blocking. The network capacity blocking cannot be solved by using more regenerator equipment: if no routes with available channels exist between two nodes, neither wavelength conversion nor regenerator equipment will allow the provisioning of a lightpath between those nodes and consequently the request will be blocked.

The use of a Linear QoT estimator allows us to model the Linear IA-RWA-RP problem as an ILP (Integer Linear Programming) combining a node-link formulation with the concept of transparent semi-lightpath (in the reminder, referred to as semi-lightpath). A semi-lightpath is an optical signal traversing a sequence of fiber links without going through any signal regeneration and any wavelength conversion. Note that this definition slightly differs from the definition used in [17], where the semi-lightpath corresponds to a lightpath with the wavelength continuity constraint relaxed. Then, a lightpath is implemented as a chain of semilightpaths, requiring one regenerator where each semilightpath ends (but the last). We emphasize as a merit of this model its simplicity which makes possible to obtain optimal solutions for the Linear IA-RWA-RP problem in medium size networks with 16 wavelengths per fiber. The IA-RWA-RP problem is clearly NP-hard, since it contains the RWA NPhard problem [18] as a special case. For planning larger networks, we present two effective heuristic algorithms: the Lightpath Segmentation (LS) and the 3-Step heuristics, being the latter specially designed to guarantee zero signal regeneration blocking. We also provide a lower bound to the regenerator equipment cost which is measured in total number of regenerators placed in the network. Note that our aim is to reduce the overall number of devices and not the regenerator sites which may contain more than one device. However, it is possible to constraint the problem setting as an input parameter the set of nodes which are able to host regenerator equipment.

On the contrary, an ILP approach for the Non-Linear IA-RWA-RP problem which adopts a Non-Linear QoT is impracticable. For this reason, we propose an efficient heuristic called IRP (Iterative Regenerator Placement). The heuristic is based on the 3-Step heuristic, and includes an iterative regeneration placement phase. In each iteration, a new regenerator is added, and the method re-optimizes the wavelength assignment, with the aim of minimizing the interferences between lightpaths. The result is a fast and effective search in the solution space.

An extensive battery of tests is included in this paper. The tests have been carefully selected, so that they correspond to networks which allow solutions with zero network capacity blocking in the linear case. Then, we are able to fairly assess both the signal regeneration blocking and the regenerator cost of the provided solutions. As a contribution of this paper, we also investigate how the regeneration equipment cost increases for different network sizes. For this, we define a normalized network size factor, which captures the relative length of the network links with respect to the longest signal propagation without regeneration.

The rest of the paper is organized as follows. In Section II we review the related work. Section III presents our investigations in the Linear IA-RWA-RP problem, while Section IV focuses on the Non-Linear IA-RWA-RP variant. Section V presents the obtained results. Finally, Section VI concludes the paper.

II. RELATED WORK

The first studies on the IA-RWA problem for translucent optical networks propose to divide the optical core network into several islands of transparency or optically transparent domains [19], [20]. An island consists of a part of the physical topology in which any lightpath can be established without intermediate signal regeneration. If a connection traverses several islands, the island boundary nodes carry out the signal regeneration. The same idea has been employed in some recent works such as [21]. The problem of this approach is its low scalability. Only the nodes on the islands' borders can host a regenerator and, in order to minimize the number of regenerators, these islands must be defined ensuring a minimum overlapping. Any change in the network (a failure, an upgrade, etc.) may require a reorganization of the islands and a re-placement of the majority of the regenerators.

An alternative approach called sparse regenerator placement is being studied in [22]-[39]. In this case any node can host in principle a regenerator (not only the nodes on the islands' border) and a regenerator placement algorithm defines the subset of nodes that actually need a regenerator: in the end they are deployed sparsely in the network. This is also the approach considered in this paper.

The majority of the proposals for regenerator placement in translucent networks deal with the IA-RWA and the regenerator placements problems separately, as if they were two different phases of the network planning. Commonly, the regenerator placement is solved first, producing a network for which the IA-RWA problem is then addressed. This strategy has been followed in [25]-[29] for static traffic, and in [22]-[24], [30]-[33] considering dynamic traffic. Other works have been presented which focus on minimizing the number of nodes with signal regeneration capability in the network, guaranteeing a certain degree of connectivity [30], [34], [35].

In [23], [24], [32] the authors study the regenerator placement and the regenerator allocation problem. The former plans the number of regenerators to be placed in each node assuming a non-deterministic traffic demand. The latter, allocates the idle regenerators to the dynamic arrivals of new connection requests. Four offline algorithms for the sparse regenerator placement and two dynamic schemes for the regenerator allocation are proposed. The regenerator placements are based on static network states and merely empirical considerations: either placing the regenerators considering the maximum transmission reach of a transparent path, in the most central nodes, in the nodes with the higher number of links, or in those with the higher loads. The regenerator allocations called fragmentation and trace back reorganize how the regenerator equipment in each node is assigned taking into account the current network state. The placement-allocation process is investigated also in [31], [33]. In [31], a heuristic algorithm is firstly applied for the regenerator placement; the nodes crossed by the majority of the shortest-paths host the regenerators. Then, the IA-RWA and the regenerator allocation problems assuming dynamic traffic are solved using a two-dimensional Dijkstra algorithm. In [33], the authors investigated the impact of the physicallayer information inaccuracy on the efficiency of a regenerator allocation technique.

In [26] a Mixed ILP (MILP) formulation and a sequential heuristic algorithm based on the *K*-least-wavelength-weight-path routing are proposed to solve the IA-RWA problem. The objective of both schemes is the maximization of the number of established connections which means that a connection blocking can occur. In principle, any node can host a regenerator but the total number of regenerators is an input value and it is not minimized.

In [34] the authors studied the problem of minimizing the number of nodes equipped with signal regeneration giving the guarantee that at least k end-to-end connections are always accepted. To this end, they suggest a two-step algorithm which is compared to a k-coverage algorithm for mobile and ad hoc wireless networks. The authors in [30], [35] propose a regenerator placement based on the greedy algorithm from [36] to get full-connectivity and two simple RWA algorithms based on shortest path and first-fit wavelength assignment. One model considers the worst-case physical transmission penalties, while the other model takes into account the current network status in order to obtain the physical impairments.

In contrast to previous works, in this paper we study how to solve the regenerator placement and the IA-RWA problems

jointly, which have been previously studied only in [25], [29], [38], [39].

In [25] a sequential algorithm called LERP (Lightpath Establishment with Regenerator Placement) is proposed. This algorithm minimizes simultaneously the number of rejected traffic demands and the number of required regenerators. In [27], [28] the authors continue their study. They investigate the impact of deploying in-line gain equalizers in terms of the number of required regenerators. Moreover, they propose two new wavelength assignment strategies to employ in the LERP algorithm. These proposals use a Linear QoT model.

In [29] the Non-Linear variant of the IA-RWA-RP problem is addressed. The algorithms proposed divide the problem into three consecutive phases. Given a set of lightpaths demands (i.e. the traffic matrix), the first phase finds those lightpaths that cannot be served transparently by any of the k-shortest available paths. An ILP model and four different heuristic algorithms are proposed to decide how to split all these nontransparent lightpaths into a sequence of transparent lightpaths and place the regenerators accordingly. The result of this phase is a transformed traffic matrix. In the second phase, an IA-RWA algorithm is applied to route the transformed traffic matrix using an ILP formulation. If some connections cannot be served, a third phase tries to reroute them using the remaining network resources. However, at the end of all these phases, some connections can be still blocked due to unacceptable QoT performance or the lack of free resources.

In [38] a heuristic algorithm for the Linear IA-RWA-RP problem is proposed. The algorithm aims at minimizing both the number of required regenerators and the number of regeneration sites in the network. The RP problem is based on a-priori choice of potential regeneration sites considering the most restrictive lightpaths whose quality level is checked without considering the impairments provoked by the neighboring lightpaths.

In [39] the problem of IA-RWA-RP with the traffic grooming is studied. The authors propose a heuristic algorithm which decouples the placement of regenerators and the dimensioning of the electronic grooming equipment. The regenerator placement sub-routine is based on shortest path routing and first-fit wavelength selection, both performed within an auxiliary reachability graph which addresses the impairment constraint just as a limit of physical hops.

In this article we deal both with the Linear and Non-Linear versions of the IA-RWA-RP problems. For the Linear case, we propose a novel ILP formulation, yielding optimal solutions to the problem and being the first one to achieve this objective as far as the authors know. The simplicity of the model allows optimal solution for small/medium scale non-trivial problems. For larger networks we also present a regenerator cost lower bound, and two novel heuristics. These heuristics have shown very good scalability properties, and accurate results, outperforming previous proposals. In addition a heuristic algorithm is proposed for the Non-Linear case, also outperforming previous proposals.

III. LINEAR IA-RWA-RP PROBLEM

A. Linear QoT estimator

In this section we make use of a Linear QoT estimator. We recall that a Linear QoT computes the O factor of a lightpath over-estimating the effects of the non-linear impairments and accumulating them to the linear ones. The fact that the Linear QoT of a lightpath does not take into consideration the effects of the other lightpaths simplifies considerably the problem formulation. In the following we provide a formal ILP model for the Linear IA-RWA-RP problem and two heuristics. Although such methods are based on a generic Linear QoT, in the performance evaluation we use the Q-Personick factor [40]. The Q-Personick factor takes into account both linear and non-linear effects. The measure of linear effects in the Q factor computation is the Optical Signal-to-Noise Ratio (OSNR). The semi-lightpath OSNR can be calculated considering the OSNR across each of the elementary optical system components (such as the fiber spans and the nodes) along the semi-lightpath and by combining the partial results. The non-linear effects are incorporated into the model by means of signal degradation factors determined by experimental analysis which account for all amplifiers (both boosters and in-line amplifiers) along the semi-lightpath. For more details on the Q factor calculation, we refer to [9].

B. Problem Formulation

In this section we present an exact ILP formulation (named as OptILP in the reminder) which solves optimally the Linear IA-RWA-RP problem. The input parameters are the physical topology, the lightpath demands and the physical impairments. Let N be the set of nodes in the network, E the set of unidirectional fiber links, and W the set of wavelengths in each link. We assume that all fibers in the network have the same number of wavelengths. We denote as a(e) and b(e) the initial and ending nodes of fiber $e \in E$. We also denote $\delta^+(n)$ and $\delta^-(n)$ the set of fibers initiated and ending at node $n \in N$ respectively. T denotes the set of lightpath demands, being T_{sd} , the number of lightpaths to be established from node s to d, $s,d \in N$.

Our formulation is based on the concept of semi-lightpath. As a pre-processing step, the set of valid semi-lightpaths $P_{\mathcal{Q}}$ is calculated in the network. They consist of the set of all the paths which are valid considering the physical impairments. Note that the set of semi-lightpaths forms a reachability graph [37][39]. The model presented in this paper uses the semi-lightpaths as the links in a node-link formulation of the problem. We assume that regenerators are capable of wavelength conversion. The decision variables of the problem are:

⁻ x_{pwsd} = {0,1}, $p \in P_Q$, $w \in W$, $s,d \in N$. x_{pwsd} takes the value 1 if the path p in P_Q uses the wavelength w for carrying one lightpath of the demand from s to d.

 $⁻t_{sd}$ = {0,1,2,...}, s,d∈N. Number of lightpaths carried from node s to node d.

The problem formulation is given by (1a)-(1d):

$$\min\left(\sum_{p \in P_Q, w \in W, s, d \in N} x_{pwsd} - \sum_{s,d \in N} t_{sd}\right) + M \cdot \left(\sum_{s,d \in N} (T_{sd} - t_{sd})\right)$$
(1a)

subject to:

$$\sum_{\substack{p \in \delta^+(n) \\ w \in W}} x_{pwsd} - \sum_{\substack{p \in \delta^-(n) \\ w \in W}} x_{pwsd} = \begin{cases} t_{sd} \text{ if } n = s \\ -t_{sd} \text{ if } n = d, \ \forall n, s, d \in N \end{cases}$$

$$0 \text{ otherwise}$$

$$(1b)$$

$$\sum_{p \in Q_e, s, d \in N} x_{pwsd} \le 1, \forall e \in E, w \in W$$
 (1c)

$$t_{sd} \le T_{sd}, s, d \in N \tag{1d}$$

The objective function (1a) consists of two elements. The first component aims to minimize the number of regenerators to be placed. The number of optical regenerators required by a lightpath is given by its number of semi-lightpaths minus 1. Then, the total number of regenerators is given by the total number of active semi-lightpaths, minus the number of carried lightpaths. The second part of the objective function deals with the minimization of the number of blocked lightpath requests. By using a constant M high enough it is possible to set the blocking minimization as the dominating criteria in the optimization (e.g. M=|W||N|). Then, a solution which carries more traffic would be always preferred whatever amount of extra signal regenerators requires. Constraints (1b) are the flow conservation constraints for the link-flow formulation. Constraints (1c) avoid the wavelength clashing, i.e., a wavelength in a fiber can be used only once. We denote as Q_e to the set of semi-lightpaths which traverse the fiber e. Finally, constraints (1d) state that carried traffic is limited by the offered traffic.

Note that this model can be easily modified adding a *placement constraint*; in such a case, only a subset N_r of the nodes is allowed to be equipped with regenerator/converter devices. The constraint (1e) introduces this into the model:

$$x_{med} = 0, \forall p \in P_O, b(p) \neq d, w \in W, s \in N, d \notin N_r$$
 (1e)

In (1e), the semi-lightpaths ending in a node without regenerator equipment capability can only be active if they are the last semi-lightpath of the lightpath. That is, for a semi-lightpath p, its ending node, which we denote as b(p), does not need to belong to the subset N_r when b(p) is the ending node of the lightpath.

C. Lightpath Segmentation Algorithm

A lightpath segmentation (LS) algorithm is an ILP-based algorithm which applies the concept of semi-lightpaths introduced in Section I. On the contrary to the ILP formulation (OptILP) presented in Section III.B, which makes

use of pre-computed semi-lightpaths that are composed into lightpaths, the idea behind LS is to start with pre-defined end-to-end paths which are then decomposed onto transparent semi-lightpaths. Besides, we must highlight that OptILP considers the set of all the existing valid semi-lightpaths obtaining an optimal solution, while LS calculates a reduced set of candidate paths to limit the complexity of the formulation and, thus, a heuristic solution is found. To formulate the problem we use a similar notation as in Section III.B.

We assume that each lightpath that is established in the network follows an explicit routing path. Accordingly, as a pre-processing step, a set of candidate paths L_{sd} (e.g., k shortest paths) is calculated for each pair of nodes $s,d \in N$, $s \neq d$. Let L denote the set of all paths.

Let L^{reg} and L^{noreg} denote, respectively, a subset of paths requiring regeneration at some intermediate node(s) and a subset of paths with no regeneration required; $L=L^{reg} \cup L^{noreg}$. For paths $p \in L^{reg}$, which are not valid considering the physical impairments, a segmentation procedure is performed in the pre-processing step to decompose them on valid semilightpaths. We perform such a procedure iteratively by segmenting a path on a number of sub-paths and by checking the physical impairment validity of each sub-path. Since path p may be segmented in a number of ways, we obtain a set of candidate segmentations R_p . Each segmentation corresponds to a sequence of valid semi-lightpaths composing path p. In order to limit the size of R_p , in our implementation of the segmentation procedure, we begin the search with the lowest number of sub-paths (i.e., two) that divides a given path and increment their number until a valid segmentation is found.

From combinatorial analysis, at the worst-case, the number

of segmentations per path is
$$R = \left(\left\lceil \frac{\delta - 1}{2} \right\rceil \right)$$
, where δ is the

length (in hops) of the longest path in L^{reg} . Concurrently, the worst-case complexity of the segmentation pre-processing step is bounded by O(S|L|), where $S = \sum_{k=1}^{\delta} k \binom{\delta-1}{k-1} = (1+\delta)2^{\delta-2}$ corresponds to the number of

QoT validations (one per each semi-lightpath).

Having calculated L and R_p , where $p \in L^{reg}$, in the preprocessing step, the LS problem concerns the selection of path p from the set of candidate paths L for each lightpath request and, if $p \in L^{reg}$, the selection of a segmentation of this lightpath from the set of candidate segmentations R_p . Concurrently, the RWA constraints, such as wavelength continuity, flow conservation, wavelength capacity, etc., have to be satisfied for all established lightpaths in the network. We continue formulating the optimization problem. First, the set of problem coefficients and constants coming from the preprocessing step are as follows:

- l_{pr} , $p \in L$, $r \in R_p$. Number of transparent segments (semi-lightpaths) on path p in L under segmentation r in R_p .

- δ_{ep} , $e \in E$, $p \in L$. Coefficient which is equal to 1 if link e belongs to path p, and equal to 0 otherwise.
- η_{elpr} , $e \in E$, $l \in \{1, ..., l_{pr}\}$, $p \in L^{reg}$, $r \in R_p$. Coefficient which is equal to 1 if link e belongs to segment l of path p under segmentation r, and equal to 0 otherwise.
- M. Big constant number used as a weighting coefficient to give a priority to the blocking objective over the regenerator usage objective; to achieve it, it is enough to have $M = |L^{reg}| \cdot |W| \cdot (\max\{s_{pr}: p \in L^{reg}, r \in R_p\} - 1) + 1$.

The problem decision variables are the following:

- $x_{sd} \in Z_+$, $s,d \in N$. Number of not-accepted lightpath requests from node *s* to node *d*.
- $x_p \in \mathbb{Z}_+$, $p \in \mathbb{L}$. Number of accepted lightpath requests that follow path p.
- $x_{pw} \in \{0,1\}$, $p \in L$, $w \in W$. x_{pw} is equal to 1 if wavelength won path p is assigned to a lightpath (in case if $p \in L^{noreg}$) or to the first semi-lightpath (if $p \in L^{reg}$), and equal to 0 otherwise.
- $-x_{pwrl} \in \{0,1\}, p \in L^{reg}, r \in R_p, w \in W, l \in \{1,...,l_{pr}\}. x_{pwrl} \text{ is}$ equal to 1 if semi-lightpath l on path p under candidate segmentation r has assigned wavelength w, and equal to 0 otherwise.

The formulation is described in (2a)-(2f):

minimize
$$\sum_{p \in I^{reg}} \sum_{w \in W} \sum_{r \in R_{-}} (l_{pr} - 1) x_{pwr1} + M \sum_{s,d \in N} x_{sd}$$
 (2a)

subject to:

$$\sum_{p \in L_{sd}} x_p + x_{sd} = T_{sd}, \qquad \forall s, d \in N$$

$$\sum_{w \in W} x_{pw} = x_p, \qquad \forall p \in L$$
(2b)

$$\sum_{w \in W} x_{pw} = x_p, \qquad \forall p \in L$$
 (2c)

$$\sum_{r \in R_n} x_{pwr1} = x_{pw}, \qquad \forall p \in L^{reg}, w \in W$$
(2d)

$$\sum_{w \in W} x_{pwr(l-1)} = \sum_{w \in W} x_{pwrl}, \qquad \forall p \in L^{reg}, r \in R_p, l = 2...l_{pr}$$
 (2e)

$$\sum_{p \in L^{noreg}} \delta_{ep} x_{pw} + \sum_{p \in L^{reg}} \sum_{r \in R_p} \sum_{l=1...l} \eta_{elpr} x_{pwrl} \le 1, \qquad \forall e \in E, w \in W$$
 (2f)

The objective function (2a) consists of two components. The first one counts the number of regenerators which depends on the number of semi-lightpaths that compose the lightpaths requiring signal regeneration. The second component represents the number of blocked lightpath requests. The constant M is assumed to be big enough so that the blocking objective dominates the optimization. Constraints (2b) are the traffic constraints, which mean that either the offered connection requests are distributed over candidate paths or they are lost. Constraints (2c) and (2d) are the wavelength assignment constraints both for the lightpaths that do not require the signal regeneration and for the lightpaths that are composed of semi-lightpaths. In particular, in (2d) the wavelength of the first semi-lightpath together with a convenient segmentation of the corresponding lightpath is determined. Concurrently, constraints (2e) are the flow conservation constraints which assign wavelengths to consecutive semi-lightpaths under the assumption that the regenerative nodes are capable of the wavelength conversion. Eventually, constraints (2f) represent the wavelength capacity constraints.

The number of variables and constraints of formulation (2) is upper-bounded, respectively, by $|N|^2 + |L| + |L||W| + R \partial |L^{reg}||W|$ and $|N|^2 + |L| + |L^{reg}||W| + R\delta |L^{reg}| + |E||W|$. Since optimization problem (2) is a variant of the RWA problem (to see it, consider $L^{reg} = \emptyset$), the problem is NP-hard. Nevertheless, under the condition that the sets of candidate paths and candidate segmentations are not large, the algorithm performance is satisfying, even for larger network instances, as shown in the results section.

D. 3-Step Heuristic

The second heuristic algorithm proposed consists of three consecutive steps.

Step 1 (Routing): An instance of the Integral Multicommodity Flow Problem [41] is optimally solved for the network. In the Integral Multicommodity Flow formulation, each link represents a fiber in the original network, with a capacity given by |W|. The flows to allocate are the lightpaths. Each carried lightpath occupies a capacity of one in each traversed fiber. The purpose of this formulation is to find a route for every lightpath demand without considering either the wavelength assignment or the physical impairments. The objective function is set to (i) minimize the lightpaths blocked, and (ii) among the solutions with the minimum blocking, search for the one minimizing the average number of physical hops of the carried lightpaths.

Step 2 (Wavelength assignment and converter placement): The carried lightpaths from the previous stage are sequentially processed. For each lightpath, a first fit wavelength assignment is carried out. When wavelength continuity is not possible, regenerators are used as wavelength converters. We use the first wavelength which allows carrying the lightpath using one regenerator. If that is not possible, the same is applied for 2, 3 etc. regenerators, till a solution is found.

Step 3 (Regenerator placement): The set of lightpaths produced from previous iterations are sequentially processed. The Q factor of each lightpath is evaluated. If its Q factor value is below the QoT threshold set, it is trivially split into the minimum number semi-lightpaths needed, placing the regenerators in the appropriate nodes.

Step 1 of the algorithm, produces the set of lightpaths to be carried, minimizing the network capacity blocking. After that, Step 2 and 3 of the algorithm heuristically search for the minimum number of regenerators which solve the wavelength clashing and signal degradation issues. However, note that no

blocking exists in both last steps. Therefore, the 3-Step Heuristic optimally minimizes the network capacity blocking, and guarantees a zero signal regenerator blocking.

Finally, while the integral version of the multicommodityflow problem used in Step 1 is known to be NP-hard, its complexity has shown to be acceptable for the network sizes of interest, as shown in the results section.

IV. NON-LINEAR IA-RWA-RP PROBLEM

A. Non-Linear QoT estimator

For the Non-Linear IA-RWA-RP problem we make use of a Non-Linear QoT estimator called Q-NL. Q-NL factor explicitly considers linear and non-linear effects. The main impairments included in this *Q* factor, and modeled according to the references, are: Amplifier Spontaneous Emission (ASE) [11], Intrachannel Crosstalk (XT) [11], Cross-Phase Modulation (XPM), Self-Phase Modulation (SPM) [12], [13], Four Wave Mixing (FWM) [13]-[15] and Polarization Mode Dispersion (PMD) [16]. Chromatic Dispersion (CD) is considered completely compensated by the transmission system.

B. Iterative Regenerator Placement Heuristic

In the Non-Linear IA-RWA-RP problem, the interferences between lightpaths are included explicitly in the QoT estimation. In this context, the Q factor of a semi-lightpath depends on the existence of other semi-lightpaths with common links and common nodes. To address this problem we propose the Iterative Regenerator Placement algorithm (IRP). It consists of the following steps:

Step 1 (*Routing*): The routing of each lightpath is conducted as in the Step 1 of the 3-Step algorithm.

Step 2 (*Converter placement*): Wavelength converters are placed as in the Step 2 of the 3-Step algorithm.

Step 3 (Iterative wavelength re-assignment and regenerator placement): The input parameters to this step are the semilightpaths defined by the already placed converters/regenerators. Note that the wavelength assignment obtained in Step 2 is not used as an input parameter.

- Wavelength assignment: The formulation (3) is executed to assign a wavelength to each semi-lightpath, minimizing an estimation of the noise variance caused by non-linear impairments.
- *Validity check*: If all the semi-lightpaths have a *Q* factor within the valid range, the algorithm ends.
- Regenerator placement: The semi-lightpath with the worst Q factor estimator is split into two semi-lightpaths by placing one regenerator. Let n be the initial node of the original semi-lightpath. The regenerator is placed at node n, so that (i) the semi-lightpath from n to n is Q-valid, (ii) but if the regenerator was placed at node n, one link further from n, the resulting semi-lightpath from n to n

would not be Q-valid.

Step 1 of the algorithm routes the lightpath demand. If lightpath blocking occurs at this stage, that would be network capacity blocking which is not solvable by placing regenerators. Further steps place regenerators till all the semilightpaths are QoT-valid.

The wavelength assignment step is obtained by solving the following ILP:

$$\min \sum_{p \in P_a} S_p \tag{3a}$$

subject to:

$$\sum_{w \in W} x_{pw} = 1, \forall p \in P_a \tag{3b}$$

$$\sum_{p \in P_a \cap Q_e} x_{pw} \le 1, \forall e \in E, w \in W$$
(3c)

$$\sum_{e \in p} \begin{pmatrix} s_{XT,b(e)}^{2} \sum_{p'|b(e) \in p'} x_{p'w} + \\ s_{XPM-1,e}^{2} \sum_{p \in P_{a} \cap Q_{e}} (x_{p'w-1} + x_{p'w+1}) \\ + s_{XPM-2,e}^{2} \sum_{p \in P_{a} \cap Q_{e}} (x_{p'w-2} + x_{p'w+2}) \\ + c_{FWM} - S_{p} \end{pmatrix} \leq \sigma_{pw,MAX}^{2} + B(1 - x_{pw}) (3d)$$

The formulation assigns one wavelength to each one of the semi-lightpaths already defined at this step of the algorithm, represented by set P_a . Constraints (3b) set that one wavelength is assigned per semi-lightpath. The wavelength clashing constraints are defined in (3c). Constraints (3d) include the effects of the non-linear impairments. Left hand side of (3d) is an estimation of the noise variance suffered by semi-lightpath p if assigned wavelength w, caused by: (i) XT from other semi-lightpaths with common nodes, and (ii) XPM and (iii) FWM caused by other semi-lightpaths with common links. In its turn, $\sigma_{pw,MAX}^2$ represents the maximum noise variance related to non-linear impairments that semi-lightpath (p,w) could accept while maintaining the required QoT. Worst-case variances s^2 and c_{FWM} , and maximum acceptable variance $\sigma_{pw,MAX}^2$ are calculated according to [11]-[15]. The variables $S_n \ge 0$ are slack variables to permit that some semilightpaths exceed the accepted noise variance. The sum of these slack variables is the figure of merit to optimize (3a).

V. RESULTS

This section collects and analyzes extensive results obtained for validating the algorithms proposed, under different testing scenarios. As a comparison, we also provide the lightpath blocking and regenerator cost performances calculated by the LERP algorithm proposed in [25] for the Linear IA-RWA-RP case, and the PH-ILPmax algorithm proposed in [29] for the

Non-Linear IA-RWA-RP case. The latter algorithm provides the best performance among the family of algorithms proposed

TABLE I INFORMATION ON THE TOPOLOGIES TESTED

INFORMATION ON THE TOPOLOGIES TESTED								
	Internet2	NSFNET	NOBEL- EU					
Reference	[44]	[45]	[46]					
Nodes	9	14	28					
Unidirectional links	26	42	82					
Average in degree (link)	1.44	1.50	1.46					
Average link length β=β ^{MIN} (km) (O-Personick)	531.3	432.4	292.2					
Average link length $\beta = \beta^{MAX}$ (km) (Q-Personick)	1806.4	1081	1043					
Ratio maximum/average link length (>1) (Q-Personick)	1.6	2.6	2.5					
Average link length β=β ^{MIN} (km) (O-NL)	425.0	324.3	-					
Average link length $\beta = \beta^{MAX}$ (km) (Q-NL)	2019.0	1189.0	-					
Ratio maximum/average link length (>1) (Q-NL)	1.6	2.6	-					

in [29]. All the algorithms have been implemented in MATLAB code, integrated and tested in the MatPlanWDM tool [42], which interfaces with the TOMLAB/CPLEX solver [43].

A. Testing scenarios

Three reference network topologies, together with their corresponding reference traffic matrices, are used in our study: Internet2 [44], NSFNET [45], and NOBEL-EU [46]. Table I summarizes some major data from these topologies. All the nodes are allowed to host regenerators. The number of wavelengths per fiber tested is $W \in \{8, 16\}$ for medium-sized networks Internet2 and NSFNET, and $W \in \{80\}$ for the NOBEL-EU test.

As commented previously, in this paper, we employ the Q-Personick factor [40] implemented as in [9], as a representative of the Linear QoT estimators and the Q-NL estimator based on analytical models for the Non-Linear QoT case. We recall that a Q factor estimates the quality of optical signal along a transparent semi-lightpath, i.e., the segment of a lightpath comprised between two regenerators.

The optical transmission system parameters considered for the Q-Personick are the same as in [9], and they are shown in Table II. We assume spans of standard single mode fiber (SSMF). The threshold on the acceptable Q factor value is equal to 17 dB. As a result, the maximum link length which is valid according to the Q threshold is 2688 km.

The Q-NL assumes the link and node architectures proposed in [47] and [11], respectively, with spans of standard single mode fiber (SSMF) under-compensated with dispersion compensation fibers (DCF) to a value of 30 ps/nm·km to diminish the non-linear effects. An appropriate post-compensation module in the end of the link compensates the accumulated dispersion. The transmission parameters used are depicted in Table II. The same threshold (17 dB) on the

acceptable Q factor is used to validate the QoT of a semi-lightpath. The resulting maximum link length considering

TABLE II
TRANSMISSION SYSTEM PARAMETERS

Parameter	Q-Personick	Q-NL		
Transmitter Bit Rate	10 Gbps	10 Gbps		
Modulation Type	-	NRZ-OOK		
Grid Spacing	-	50 Ghz		
Maximum Span Length	85 km	85 km		
Input Power to SSMFs	3 dBm	3 dBm		
Input Power to DMFs	-	-4 dBm 0.23 dB/km 0.5 dB/km 17 ps/nm·km		
SSMF Attenuation Parameter	0.23 dB/km			
DCF Attenuation Parameter	-			
SSMF CD Parameter	-			
DCF CD Parameter	-	-80 ps/nm·km		
Noise Figure of in-line and pre- amplifiers	5 dB	5 dB		
Noise Figure of boosters	6 dB	6 dB		
PMD Parameter	-	$0.1 \text{ ps}/\sqrt{km}$		
Switching Power Attenuation	13 dB	as in [11]		
Switch Crosstalk Ratio	-	32 dB		

worst-case non-linear impairments (links totally populated with lightpaths) is in the order 3000 km, with slight variations between different topologies and number of wavelengths per fiber.

In our tests, three traffic loads are considered: low, medium and high ($\rho \in \{0.4, 0.7, 1\}$). Given a network topology, a reference traffic matrix for that topology T^{BASE} (measured in any arbitrary traffic units), a number of wavelengths per fiber W and a traffic load factor ρ , the lightpath demand matrix is calculated as follows. First, we calculate the maximum lightpath demand matrix T^{MAX} . This is done by finding the maximum value α^{MAX} for which the lightpath demand matrix in (4) admits a feasible routing solution for an instance of the integral multicommodity flow problem (built as in the Step 1 of the 3-Step algorithm). This guarantees that the lightpaths in T^{MAX} can be fully carried by the network with a zero percent of lightpath blocking if a sufficient number of regenerators are used

$$T^{MAX} = round \left(\alpha^{MAX} \cdot T^{BASE} \right) \tag{4}$$

The maximum lightpath demand matrix, T^{MAX} is associated to load $\rho = 1$. Let us denote S_T the total number of lightpaths in T^{MAX} , $S_T = \sum_{s,d \in N} T^{MAX}_{sd}$. Then, the lightpath demand matrices

 T^{ρ} at other network loads ρ , are calculated by finding the factor $\alpha < \alpha^{MAX}$ for which its associated lightpath demand matrix has a volume equal to ρS_T (or its closest integer value).

$$T^{\rho} = \begin{cases} round \left(\alpha \cdot T^{BASE} \right), \text{ so that } \alpha \leq \alpha^{MAX}, \\ \sum_{s,d \in \mathbb{N}} T^{\rho} = round(\rho S_{T}) \end{cases}$$
 (5)

The traffic normalization designed in our tests implies that all the planning instances admit a solution with a zero percent of network capacity blocking. We are also interested in studying the effects of the network link lengths on the regenerator equipment cost planned. For each network topology, we follow this sequence of steps:

- 1) We calculate the maximum factor β^{MAX} so that after multiplying the length of all the links of the network by β^{MAX} , the longest link has a Q factor value just equal to the acceptable detection threshold (17 dB). Note that values $\beta > \beta^{MAX}$ are not considered, since they could require an optical signal to be regenerated in the middle of a fiber link.
- 2) We calculate the minimum factor β^{MIN} so that after multiplying the length of all the links of the network by β^{MIN} , the shortest path between every pair of nodes with the worse (lowest) Q factor still has a Q factor Q = 17 dB.
- 3) We repeat the tests for four distance factors in the network: $\{\beta^1, \beta^2, \beta^3, \beta^4\}$, where $\beta^1 = \beta^{MIN}$, $\beta^4 = \beta^{MAX}$, and we obtain β^2 and β^3 as intermediate points between the values of β^1 and β^4 .

Note that β multipliers define a sort of normalized network size. They depend on the relation between the link lengths and the QoT degradation. This latter effect is affected by technological aspects like the transceivers bit rate and modulation, or the amplification and compensation equipment installed in the network. In other words, higher β multipliers could be associated either to continental long-haul networks, or to smaller networks with e.g. shorter reach transmission technologies.

B. Results: Linear IA-RWA-RP case

Tables III and IV collect the testing results for the networks Internet2 and NSFNET. In both cases, the tests has been conducted for $W \in \{8, 16\}$ wavelengths, three load levels $\rho \in \{0.4, 0.7, 1\}$ and 4 distance factors $\beta \in \{\beta^1, \beta^2, \beta^3, \beta^4\}$. In the linear case, five regenerator placement methods are compared: (i) OptILP, the exact formulation presented in Section III.A, (ii) 3-Step heuristic in Section III.C, (iii) LS heuristic in Section III.B, and (iv) the LERP algorithm proposed in [25]. The LERP algorithm has been executed ranging different values of a set of specific parameters that tune how the solution space is heuristically explored. The results shown in this paper correspond to those parameters which provided the best performances. We reproduce their values to allow the results in this paper to be repeatable: (i) the number of shortest paths computed associated to each demand is set to 4, (ii) the LERP black list size is set to 100, and (iii) the number of permutations performed in the demand set is 10000. The reader should refer to [25] for more details on the operation of the LERP algorithm. For the LS algorithm the set of candidate paths is calculated as k shortest paths between every pair of nodes according to the physical distance. When the value of k is selected, we must take into account that using more candidate paths avoids the problem of network capacity blocking, but it increases the algorithm complexity. Therefore, we execute several experiments for each topology scenario and find the lowest number of candidate paths that allows to reach the zero-blocking objective, in particular, k = 3, k = 5, and k = 2, respectively, for Internet2, NSFNET, and NOBEL-ELI

In Tables III and IV we provide the information related to the regenerator equipment cost and the execution time of the algorithms. The regenerator cost is given as the average number of regenerators that a carried lightpath needs (in %). That is, the total number of regenerators planned divided by the carried demand volume. The lightpath blocking information is not provided for the OptILP, 3-Step, and LS algorithms since it is zero for all of them. Note that this was guaranteed for both the OptILP and 3-Step methods. Results have shown that although LS heuristic does not necessarily guarantee a zero percent of signal regeneration lightpath blocking, it provides this benefit in practice thanks to the appropriate choice of the set size of pre-defined paths. In contrast, the LERP algorithm exhibits lightpath blocking in numerous cases. The column LB provides a lower bound to the regenerator cost. It corresponds to the number of regenerators needed if each of the lightpaths was carried alone in the network using the the path with the lowest possible number of semi-lightpaths traversed. The interest on plotting the LB information in Table III, is evaluating its accuracy by comparing its value to that of the optimal cost in OptILP column. The accuracy of the LB has shown to be perfect in the Internet2 topology. For the NSFNET network, a null or small gap between the LB and the optimal cost is observed in all the low and medium load occasions. A more significant gap between the LB and the optimal cost is found in high network loads and high normalized network sizes (with a maximum of \sim 15% in the worst case). This is logical, since the lower bound assumes a shortest path routing, which is in general not possible at high load conditions.

By comparing the regenerator cost performance in OptILP with those from the heuristic algorithms, we can assess their quality for the small to medium size topologies tested. Results in Table III show that the number of regenerators required by the 3-Step and LS algorithms are equal to the results obtained by OptILP in almost all the cases. On the contrary, the LERP algorithm obtains accurate results only for the distance factor β¹. For the rest of scenarios, it needs a higher number of regenerators and/or incurs in lightpath blocking. Note that in some occasions, the regenerator cost of the LERP solution is lower than the optimum. This is because LERP has significant lightpath blocking and, therefore, requires a smaller number of regenerators. In summary, the 3-Step and LS algorithms outperform the LERP algorithm for the scenarios considered obtaining a close to optimal regenerator cost, without incurring in lightpath blocking.

Table V collects the results of the heuristic algorithms for the case of NOBEL-EU network, and W = 80 wavelengths per fiber. Given the larger number of nodes and wavelengths, it has not been possible to obtain results with OptILP. However, it is very interesting to see that in this case, both the 3-Step and LS heuristics achieve the optimum solution that minimizes

the regenerator equipment cost with a zero percent of lightpath blocking. Optimality is guaranteed since its cost equals the cost of the lower bound. The results of 3-Step and LS algorithms outperform the ones of LERP, which incurred in lightpath blocking in the instances with normalized network sizes β^3 and β^4 . In some of these situations, the LERP algorithm required a lower amount of regenerators than the lower bound. Again, this is due to the higher number of blocked lightpaths in the LERP solution, causing an overall decrease of the regeneration requirements.

Summarizing the comparison of the heuristics, the LS and 3-Step algorithms provide very similar solutions in all the occasions, very close to the optimal solutions. In Internet2 and NSFNET cases, they were more frequent the cases in which the LS algorithm provided better solutions. Finally, some interesting remarks can be made by observing the regenerators placed by 3-Step algorithm in its second and third step. It has been observed that in practically all of the cases, no regenerators were placed in step 2 of the algorithm. That means that regenerators were seldom needed for wavelength conversion purposes. Only in some problem instances at higher loads, some lightpaths (below 0.5%) changed their wavelength along the path. The uncommonness of the wavelength conversion in offline planned optical networks is supported by previous studies like [48].

By observing the values in the OptILP column and the results from the LS and 3-Step heuristics in Table III, it is possible to capture some trends in the regenerator equipment cost. It seems clear that irrespective of the network load, networks with a small normalized size $(\beta = \beta^{MIN})$ do not require regeneration equipment. After that, larger network sizes are associated to higher regeneration needs. The same happens with the network load conditions, if we measure the regenerator equipment in absolute values. However, in some occasions higher load values implied lower per-lightpath regenerator cost. We observed that this behavior is explained by how the traffic matrices are synthesized. The round operation in equation (5) adds a sort of uncontrolled effect, depending on whether the coordinates of the lightpath demand matrix corresponding to distant nodes are more frequently rounded up or rounded down at a given network load. Finally, the regeneration equipment cost showed to be significantly topology-dependent. For the maximum network factor β^{MAX} , the number of regeneration units per carried lightpath was in the order of 60% for Internet2 network, and 30% for the NSFNET and NOBEL-EU topologies. The reason for that difference can be found in the ratio between the maximum and average link lengths in the network. In Internet2 topology, the maximum link length is only 1.6 times the size of the average link length, while in NSFNET and NOBEL-EU topologies, there is a higher disparity in link lengths. Then, the average link length measured when we normalize the network at $\beta = \beta^{MAX}$ is significantly higher in the Internet2 topology (~1800 km in Internet2, and ~1000 km in NSFNET and NOBEL-EU). Consequently, the percentage of the paths that need signal regeneration at $\beta = \beta^{MAX}$ is also higher in the Internet2 topology.

Table III and IV display the execution time observed in each of the tests performed for all the algorithms. The most reduced execution times are obtained by the 3-Step algorithm whose times were *below of* 1 *second* in all the tests.

Naturally, the highest execution times correspond to the exact problem solving, in OptILP column. For Internet2, the execution times were reasonable, below two minutes in all the occasions. In the NSFNET network, execution times ranged from minutes to several hours. The longest time observed was about 80 hours. Running times are higher for a higher number of nodes in the network and a higher number of wavelengths. This is caused by the increase in the number of decision variables of the problem. The effects of the network load and the normalized network size factors are more random since they affect the performance of the branch-and-bound pruning step. As general trend, longer running times were observed for low to medium normalized network size factors.

The execution of the LS algorithm has shown to be around 1 second in all the Internet2 tests, and for NSFNET W=8. The rest of the execution times are in the order of tens of seconds, always below 2 minutes. Longer execution times seem to be associated to larger networks with normalized size factors β^3 and β^4 . However, this trend is not deterministic, since it is again affected by the performance of the branch-and-bound algorithm solving the LS formulation. Finally, the response time of the LERP algorithm is in general longer than the 3-Step and LS execution times.

C. Results Considering Non-Linear Impairments

Columns IRP and [29] of Table III show the planning results of the IRP algorithm and PH-ILPmax algorithm [29] respectively. Results show that IRP outperforms PH-ILPmaxby eliminating the lightpath blocking, in all circumstances. In the occasions in which the PH-ILPmax algorithm produces solutions with zero blocking, the regeneration cost of IRP is always equal or lower. Note that the lightpath blocking is in general quite high in the solutions found by PH-ILPmax. Moreover, in the cases tagged with "-", the PH-ILPmax algorithm is not able to find a feasible solution in its Phase 2, which stops the algorithm (see [29] for details)

The trends in the regenerator cost in the Non-Linear IA-RWA-RP problem are similar to that of the linear case. This is logical since the same principles apply in the relations among traffic, link length and signal impairments. However, the absolute numbers of regenerators do not coincide. In general, the regenerator costs associated to the solutions of the nonlinear case are significantly higher. This happens more intensely for higher network loads and higher distance factors. It is explained by the different assumptions about the physical layer, node and link models, and by how the tests are designed. In the tests, the maximum distance factor (β^{MAX}) in the network is calculated as the maximum distance of a link to be able to accommodate a one hop semi-lightpath. For this situation, the Q-Personick factor produces more pessimistic Q

values. Because of the link distance normalization performed, this produces smaller networks for distance factors β^3 and β^4 than in the Q-NL case (see the average link lengths $\beta = \beta^{MAX}$ for Q-Personick and Q-NL in Table I). The opposite situation happens for $\beta = \beta^{MIN}$. In this case, the distance factor β^{MIN} depends on the Q factor of a lightpath following the longest shortest path in the network, and now Q-NL provides more pessimistic estimations. The reason for these effects is that the Q-NL factor overestimates (with respect to the Q-Personick factor) the impairments which depend on the number of nodes traversed (like crosstalk). Then, the Q factor of those routes requiring more hops will be penalized in the non-linear case. This happens more often with high network loads (since the lightpaths may have to find routes with more hops), and with higher distance factors (since more routes are long enough to require regenerators, and the links are longer in the Q-NL factor normalization). We would like to stress that the QoT estimators were chosen as representatives suitable to validate the quality of the planning algorithms. However, the results in this paper cannot be used to directly compare the Q factor estimators between them, since they do not relay on exactly the same network model.

Finally, columns IRP and [29] in Table IV show the execution time results for both algorithms. IRP is faster than PH-ILPmax for lower load and β values, but can be substantially slower in the rest of the cases. We have performed a code profiling study to observe the causes of this effect. Interestingly, the study shows that about 90% of the time is used in the O factor computations which are required in each algorithm iteration. Recall that the number of iterations is roughly given by the number of regenerators in the final solution, which increases with network load and network size. In particular, the ILP formulation (3) required less than 1 second to execute in the majority of the cases and 10 seconds in the worst case. Therefore, the scalability of the algorithm can be greatly improved by using other QoT estimators with lower computation requirements. In its turn, the PH-ILPmax algorithm makes a much lower use of the O factor computation function, and thus is not so intensely affected by this issue.

VI. CONCLUSIONS

This paper investigates the offline network planning and regenerator placement in translucent optical networks, minimizing the regenerator placement cost. We separately define and address the Linear and Non-Linear variants of these problems. As far as the authors know, we provide the first ILP model to optimally solve the Linear IA-RWA-RP problem. Thanks to its simplicity, it is able to solve problem instances in small to medium scale networks. For larger network topologies, we present two heuristic algorithms named LS and 3-Step. In the Non-Linear case, the IRP heuristic is proposed.

An extensive battery of tests is conducted. The traffic load in the tests is normalized to fairly assess the ability of the algorithms for minimizing the regeneration cost, without producing any signal-regeneration related to lightpath blocking. The results show that the LS and 3-Step algorithms provide optimal or close-to-optimal solutions in all these tests. They outperform a previous heuristic algorithm presented [25], both in the quality of the solution found and the algorithm execution time. In addition, the 3-Step algorithm guarantees that no signal regeneration lightpath blocking is produced. Consequently, both algorithms can be used to efficiently solve the Linear IA-RWA-RP problem (e.g. selecting the best solution provided by both schemes). For solving the Non-Linear IA-RWA-RP problem, we present the IRP heuristic. This heuristic is able to provide a zero signal regenerator blocking. It outperforms in both lightpath blocking and regenerator equipment cost the previous proposals in the literature tested.

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 $TABLE\ III$ Internet 2 and NSFNET: Percentage of regenerators (and blocking)

		Internet2									NSFNET					
		Linear					Non-Linear			Linear				Non-Linear		
w	β ρ	ILP	3- Step	LS	LERP (%Block.)	LB	IRP	[29]	ILP	3- Step	LS	LERP (%Block.)	LB	IRP	[29]	
	0.4	0.0	0.0	0.0	$(0.0)\ 0.0$	0.0	0.0	$(0.0) \ 0.0$	0.0	0.0	0.0	(0.00)0.0	0.0	0.0	$(0.0)\ 0.0$	
	1 0.7	0.0	0.0	0.0	$(0.0)\ 0.0$	0.0	0.0	(0.0) 4.3	0.0	0.0	0.0	$(0.00) \ 0.0$	0.0	0.0	$(0.0)\ 0.0$	
	1	0.0	0.0	0.0	$(0.0)\ 0.0$	0.0	0.0	(0.0) 14.7	0.0	0.0	0.0	$(0.05)\ 0.0$	0.0	0.8	$(2.6)\ 0.0$	
	0.4	7.6	7.6	7.6	(3.8) 4.0	7.6	11.5	(4.0) 24.0	0.0	0.0	0.0	(0.00) 0.0	0.0	0.0	(4.3) 13.0	
	2 0.7	4.3	4.3	4.3	$(2.1)\ 2.2$	4.3	10.8	(17.9)46.1	0.0	1.1	0.0	(0.01) 1.2	0.0	1.1	(3.7) 22.2	
8	1	5.8	10.2	5.8	(0.0) 8.8	5.8	25.0	(11.4)72.1	2.5	3.3	2.5	(0.06) 0.8	0.0	13.3	$(23.7)\ 0.0$	
0	0.4	7.6	7.6	7.6	(3.8) 4.0	7.6	19.2	(8.3) 8.3	0.0	0.0	0.0	(0.00) 0.0	0.0	0.0	(0.0) 8.3	
	3 0.7	8.7	8.7	8.7	(4.3) 4.5	8.7	19.5	(12.1) 56.0	3.5	4.7	3.6	$(0.07)\ 0.0$	3.5	19.0	(3.7)40.7	
	1	26.4	26.4	26.4	(1.4) 35.8	26.4	57.3	(54.4) 36.3	13.3	13.3	13.3	(0.08) 6.3	6.6	41.6		
	0.4	23.0	23.0	23.0	(7.6) 12.5	23.0	23.0	(0.0) 23.0	2.0	2.0	2.1	(0.02) 0.0	2.0	8.3	(4.3) 43.5	
	4 0.7	34.7	34.7	34.7	$(4.3)\ 31.8$	34.7	36.9	(0.0) 39.1	16.6	17.8	17.9	(0.14) 6.9	11.9	33.3	-	
	1	61.7	61.7	61.7	(1.4)71.6	61.7	69.1	-	30.0	31.6	30.0	(0.18) 19.1	17.5	55.8	_	
	0.4	0.0	0.0	0.0	$(0.0) \ 0.0$	0.0	0.0	(0.0) 3.5	0.0	0.0	0.0	(0.00) 0.0	0.0	0.0	-	
	1 0.7	0.0	0.0	0.0	$(0.0)\ 0.0$	0.0	0.0	$(1.0)\ 18.5$	0.0	0.0	0.0	$(0.00) \ 0.0$	0.0	0.0	-	
	1	0.0	0.0	0.0	$(0.0)\ 0.0$	0.0	0.0	$(13.8)\ 0.0$	0.0	0.0	0.0	$(0.05) \ 0.0$	0.0	0.4		
	0.4	3.5	3.5	3.5	(1.7) 1.8	3.5	5.3	(16.6) 0.0	0.0	0.0	0.0	(0.00) 0.0	0.0	1.0	(3.2) 23.4	
	2 0.7	12.2	12.2	12.2	$(0.0)\ 15.3$	12.2	26.5	(18.8) 86.7	0.0	0.0	0.0	$(0.00) \ 0.5$	0.0	8.8	-	
16	1	8.5	11.4	8.5	(0.0) 13.5	8.5	25.0	(55.5) 2.2	1.2	2.8	1.2	(0.06) 1.3	0.0	22.2		
10	0.4	14.2	14.2	14.2	(1.7) 14.5	14.2	16.0	(1.8) 52.7	5.1	5.1	5.2	(0.03) 2.1	5.1	14.4	(7.8) 45.6	
	3 0.7	28.5	28.5	28.5	(0.0)42.8	28.5	64.2	(48.4) 39.3	10.5	11.7	10.6	(0.06) 11.8	10.5	40.0	(11.8) 86.8	
	1	24.2	27.1	25.7	(0.0) 43.5	24.2	60.7	(72.8) 37.0	16.4	18.1	16.9	(0.06) 17.4	8.6	50.6	_	
	0.4	39.2	39.2	39.2	(1.7) 40.0	39.2	35.7	(3.7) 48.1	13.4	14.4	13.4	(0.08) 6.7	13.4	38.1	(5.4) 54.4	
	4 0.7	63.2	65.3	63.2	(0.0) 73.4	63.2	75.5	(8.8) 138.8	23.5	28.8	24.7	(0.06) 29.5	20.5	51.1	-	
	1	55.7	57.1	55.7	(0.7)74.1	55.7	67.1	-	33.7	36.2	33.7	(0.07) 38.7	18.5	68.3	-	

$$\label{eq:table_iv} \begin{split} & TABLE\:IV \\ & \text{Internet2}\:\text{and}\:NSFNET:\:Execution\:times}\:(s) \end{split}$$

				In	ternet2			Nsfnet						
		Linear				Non-Linear		Linear				Non-Linear		
w	β ρ	ILP	3-Step	LS	LERP	IRP	[29]	ILP	3-Step	LS	LERP	IRP	[29]	
	0.4	9	< 1	< 1	< 1	8	34	503	< 1	< 1	< 1	15	38	
	1 0.7	7	< 1	< 1	< 1	10	74	3830	< 1	< 1	< 1	20	51	
	1	26	< 1	< 1	< 1	14	92	65989	< 1	2	< 1	68	117	
	0.4	1	< 1	< 1	< 1	19	21	120	< 1	< 1	< 1	17	83	
	2 0.7	1	< 1	< 1	< 1	41	25	188	< 1	1	< 1	43	91	
8	1	2	< 1	< 1	1	251	34	2925	< 1	4	2	532	117	
8	0.4	< 1	< 1	< 1	< 1	28	6	51	< 1	< 1	< 1	23	53	
	3 0.7	< 1	< 1	< 1	< 1	65	15	101	< 1	1	< 1	304	62	
	1	< 1	< 1	< 1	1	539	17	2368	< 1	3	1	1615	-	
	0.4	< 1	< 1	< 1	< 1	34	5	27	< 1	< 1	< 1	54	41	
	4 0.7	< 1	< 1	< 1	< 1	114	15	43	< 1	1	< 1	501	-	
	1	< 1	< 1	< 1	< 1	658	-	123	< 1	5	1	2205	-	
	0.4	17	< 1	< 1	< 1	47	306	1540	< 1	< 1	< 1	81	-	
	1 0.7	22	< 1	< 1	< 1	73	352	180594	< 1	36	< 1	118	-	
	1	109	< 1	< 1	6	102	372	>300000	< 1	55	8	518	-	
	0.4	3	< 1	< 1	< 1	114	93	169	< 1	1	< 1	154	311	
	2 0.7	5	< 1	< 1	< 1	2313	126	343	< 1	16	< 1	2315	-	
16	1	11	< 1	< 1	13	5261	123	202857	< 1	100	7	14164	-	
10	0.4	1	< 1	< 1	< 1	226	57	97	< 1	1	< 1	866	260	
	3 0.7	1	< 1	< 1	< 1	4828	65	12796	< 1	8	1	9121	307	
	1	5	< 1	< 1	12	11235	53	11884	< 1	83	1	16879	-	
	0.4	< 1	< 1	< 1	< 1	442	49	60	< 1	2	< 1	1292	184	
	4 0.7	< 1	< 1	< 1	< 1	5611	88	284	< 1	8	< 1	6714	-	
	1	< 1	< 1	< 1	78	12447	-	18879	< 1	28	9	22656	-	

 $\label{table v} \mbox{NOBEL-EU: Percentage of regenerators (and blocking) and execution times (s)}$

			No. R	egenerators (%	n)	7	Гіте	(s)
β	ρ	3-Step	LS	LERP (%Block.)	LB	3-Step	LS	LERP
	0.4	0.0	0.0	$(0.0) \ 0.0$	0.0	< 1	3	23
1	0.7	0.0	0.0	$(0.0)\ 0.0$	0.0	< 1	4	43
	1	0.0	0.0	$(0.0)\ 0.0$	0.0	< 1	7	61
	0.4	0.0	0.0	$(0.0) \ 0.0$	0.0	< 1	4	23
2	0.7	0.0	0.0	$(0.0) \ 0.0$	0.0	< 1	5	43
	1	0.0	0.0	$(0.0)\ 0.0$	0.0	< 1	8	61
	0.4	5.6	5.6	(3.5) 2.2	5.6	< 1	11	23
3	0.7	8.5	8.5	(2.4) 7.4	8.5	< 1	15	43
	1	7.0	7.0	(1.8) 6.1	7.0	< 1	18	60
	0.4	18.4	18.4	(8.8) 10.5	18.4	< 1	12	23
4	0.7	29.1	29.1	(7.6) 31.1	29.1	< 1	27	43
	1	24.7	24.7	$(6.3)\ 26.3$	24.7	< 1	60	61