A paths algebra framework for routing and resources assignment in EONs

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Abstract

Routing and resources assignment represent one of the major challenges for Elastic Optical Networks (EONs). In this paper we propose a general mathematical framework based on paths algebra that allows the implementation of differentiated services according to different policies. The framework enables to consider simultaneously different metrics associated to the optical links. Any number of linear and non-linear metrics can be used. In this way, Quality of Service (QoS); Quality of Transport (QoT), Quality of Network Economics (QoNE), Quality of Energy (QoEn) etc. can be optimized.

Keywords

Elastic Optical Networks (EONs); Paths Algebra; Service Level Agreement (SLA); Quality of Service (QoS); Quality of Transport (QoT); Quality of Energy (QoEn); Quality of Network Economics (QoNE).

I. Introduction

In WDM transparent networks data can be switched exclusively in the optical domain without the need of optical-electronic-optical conversion. The introduction of a robust control plane provides automatic operation, and gives raise to Wavelength Switched Optical Networks (WSON). In WSONs the optical signal is switched at the wavelength granularity. A lightpath is established by the assignment of a physical route and an available wavelength. The Routing and Wavelength Assignment (RWA) problem plays a crucial role in the dynamic network operation [1].

The RWA problem can be solved either online or offline. In an online RWA scenario, demands arrive in different moments while the network is already fully operational, and connections are established and tear down dynamically. In an offline scenario the network not in use, all links have their full capacity available and the demand matrix is known in advance. The task consists in assigning the available resources and paths to the demands during the planning phase. The RWA problem may be solved as one problem or as two separate subproblems. For the first option computational complexity becomes extremely high, and, for this reason, it is usual and useful to separate the RWA problem into two sub-problems: the routing and the wavelength assignment. Several solutions can be found in the literature [1].

In the traditional ITU-T DWDM frequency grid [2] spectrum bands are spaced by 50 or 100 GHz. Each spectrum band is able to accommodated one wavelength. A flexi-grid described in [3], divides the available optical spectrum into spectrum bands of fixed narrower spectral width. The currently proposed spectral widths are 25 GHz, 12.5 GHz and 6.25 GHz. 12.5 GHz or 6.25 GHz are most commonly found in the literature and the potential bandwidth granularity that will be adopted by the industry. Each of these spectrum bands in the flexi-grid is denominated a Frequency Slot (FS). A certain number of FSs can be jointly allocated to a connection in order to accommodate a connection requirement.

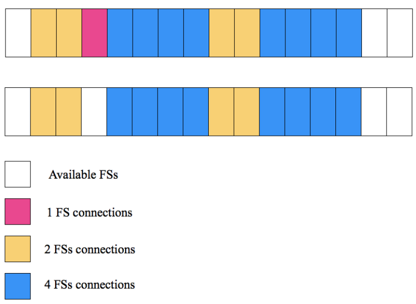
An Elastic Optical Network – EON – is based on the flexible use of the optical spectrum enabled by the concatenation of FSs into one optical channel. The flexible allocation of spectrum results in an improvement in relation to the fixed grid used in WSONs because on the one hand the resulting channels are able to serve a bandwidth requirement that is smaller than a wavelength capacity without wasting resources. On the other hand, if a demand requires more bandwidth than the wavelength capacity, more FSs may be allocated forming an optical channel able to accommodate this demand. When a large amount of spectrum slots are jointly allocated, the resulting channel may achieve high bit rates as 400 Gbps or 1 Tbps [4], [5].

In an – EON – the resources assignment refers to the allocation of spectrum resources, i.e. number of Frequency Slots – FSs. Therefore, the WSON's RWA problem becomes in an EON the routing and spectrum assignment (RSA) problem [6]. An RSA algorithm computes an end-to-end physical route and allocates a set of FSs around a central frequency. In absence of wavelength converters, RSA is subject to the spectrum continuity constraint meaning that the same FSs (represented by the FS index) in the optical signal must be assigned along all the links in the path [7]. If the spectrum continuity is not preserved, nodes must convert the central frequency along the path [4], an action that increases the complexity and cost, and should be avoided.

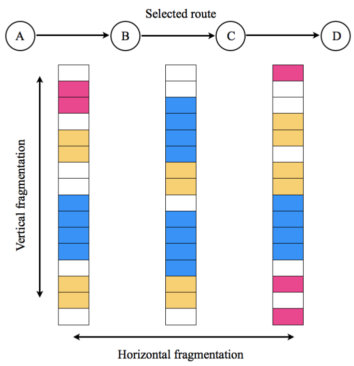
The RSA problem may be implemented in a static or in a dynamic scenario. In a static scenario the RSA algorithms are implemented in the network planning phase when the set of connection requests is known in advance. In the dynamic scenario dynamic RSA algorithms are implemented to provision connections at request arrivals. In a dynamic traffic environment, due to the real-time nature of the problem, RSA algorithms must be fast.

As mentioned for the RWA problem, online RSA algorithms can deal with the routing and spectrum assignment jointly (i.e., in one step) or separately (i.e., in two steps) [8]. In one step algorithms, routing and spectrum assignment are solved simultaneously. The drawback of one step RSA algorithms is that the problem becomes highly computational complex and time consuming. In two step algorithms, the RSA problem is decomposed into two subproblems: the routing and the spectrum assignment subproblems. The routing and the spectrum assignment are then solved separately and sequentially. The two step RSA algorithm will first compute a number of physical routes for each source-destination node pair and order them following a specific policy. Different spectrum allocation policies have been proposed in the literature. The first-fit (FF) policy [9], [10] selects the lowest index set of available contiguous FSs. In random policy one of the available sets of contiguous FSs is selected randomly [11], [12].

In a dynamic network scenario, the constant setup and release of connections can create gaps of contiguous available FSs. Authors in [8] classify the fragmentation issue according to the constraint they jeopardize. The vertical fragmentation affects the spectral contiguity constraint. It occurs when the spectral resources in a link are fragmented into various small size gaps of available contiguous FSs. The smaller the number of contiguous available FSs, the less likely these groups of available FSs would be able to serve a connection request. Figure 1-a illustrates a link before and after a connection release, and it can be seen how the link spectrum becomes fragmented. The spectrum fragmentation problem may lead to inefficient use of resources use increasing the blocking probability. The horizontal fragmentation impairs the continuity constraint and occurs when a gap of available FSs in one link does not have the same correspondent FS availability along the successive links in the path. In this case available FSs in a link may not be assigned to a connection request even though each individual link may have enough contiguous FSs to attend the demand's spectrum requirement. This fragmentation problem is also known as the misalignment of available FSs in a path's links [13]. In this fragmentation problem the smaller the number of links in a selected path the less likely the misalignment of FS availability will occur between selected links and neighboring links. An example of vertical and horizontal fragmentation may be observed in Figure 1-b.



(a)



(b)

Fig. 1. EON frequency slot fragmentation: a) release of a connection destroys the FSs contiguity; b) horizontal and vertical fragmentation.

It is out of the scope of this work to propose any new FS-aware RSA algorithm. Instead, the objective is to show how the paths algebra mathematical framework [14] can be used to map the FS-aware RSA problem in a way that different policies can be exercised and evaluated [15]. To achieve this objective this paper is organized as follows: after this brief Introduction, in Section II the FS-aware RSA problem and associated metrics is described; in Section III the paths algebra framework is presented; in Section IV the FS-aware RSA problem is mapped into the paths algebra framework; Section V provides conclusions remarks and future work proposals.

II. FS-AWARE RSA PROBLEM AND ASSOCIATED METRICS

In this section we present the FS-aware RSA problem as it has been described in [13].

Figure 2 shows 6-node, 8-link mesh network, designated as FISH network and the spectral resources are distributed as shown in Table I. In this example we assume only 12 spectrum slots on each fiber link. When a new request A – E arrives with a bandwidth requirement of 1 slot, the routing algorithm first calculates all possible routes, resulting in the five shortest paths: (i) ADE; (ii) ABDE; (iii) ABCE; (iv) ABCFE; (v) ADBCE. There are six paths from A to E in total, but path ADBCFE is omitted since we consider only $k(k = 5)$ shortest paths for each source - destination pair.

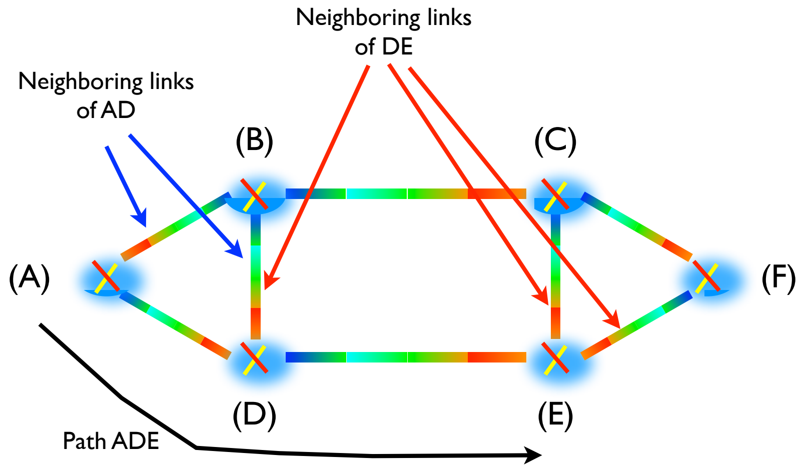


Fig. 2. Example FISH network. Adapted from [13].

TABLE I: Slots occupancy of the FISH network's links

Link	Slots											
	1	2	3	4	5	6	7	8	9	10	11	12
AB	1	0	1	1	0	0	1	1	1	0	0	0
BA	1	1	0	0	1	1	0	0	1	1	0	0
AD	1	0	0	0	1	1	1	0	0	0	0	0
DA	1	0	0	0	1	1	1	0	0	0	0	0
BC	0	0	0	1	0	0	1	1	0	0	0	0
CB	0	0	0	1	0	0	1	1	0	0	0	0
BD	0	0	0	1	1	0	0	0	1	1	0	0
DB	0	0	0	1	0	0	1	1	0	0	0	0
CE	1	0	0	0	0	0	0	0	0	0	0	0
EC	1	0	0	0	0	0	0	0	0	0	0	0
CF	1	0	1	0	0	0	0	0	0	0	0	0
FC	1	0	1	0	0	0	0	0	0	0	0	0
DE	1	0	0	0	1	0	0	0	0	0	0	0
ED	1	0	0	0	1	0	0	0	0	0	0	0
FE	1	1	0	0	0	1	1	1	0	0	0	0
EF	1	1	0	0	0	1	0	1	0	0	0	0

Figure 3-a illustrates some RSA candidate solutions for paths ABDE and ABCE. It can be seen that some of the solutions will break the contiguousness of a spectral block on one or more of the links on the current path. The cuts are considered to be the costs of the candidate solutions, since more cuts create more fragments on the candidate links of the routes. For example, the provisioning of the request on path ABCE with slot 10 will cut two spectrum blocks on links BC and CE, namely, the contiguous spectral slots 9 – 12 on link BC and spectral slots 2 – 12 on link CE. In the example in Figure 3, the assignment of slot 6 on ABDE give zero cuts; therefore, it should be the most preferred solutions in terms of spectral fragmentation awareness.

On the other hand, the provisioning of a request can also increase the misalignment of the available spectral blocks between the candidate links and their neighboring links. The optimized spectrum assignments are keeping the unused spectrum on neighboring links aligned for future requests. For example, the candidate path ABDE will change the alignment of the neighboring links as shown in Figure 3-b. If slot 6 is assigned the misalignment for the link pair DE and EF will decrease by one, since the provisioning on link DE on slot 8 reduces the misalignment by provisioning fills up the originally misaligned spectrum. Likewise if slot 11 is assigned the the misalignment for the link pair DE and EF will increase by one as the commonly available spectrum is decreased by one slot. The increased misalignment in a network is considered as an additional cost for future lightpath provisioning and spectrum defragmentation. Therefore, among all candidate solutions, the algorithm should minimize the misalignment cost as well.

Overall, the cuts and misalignment increase costs are the two proposed metrics in the lightpath provisioning process. However, the minimal-cut solutions may conflict with the minimal misalignment increase solutions. Therefore, the fragmentation-aware RSA algorithms should take into account both costs jointly.

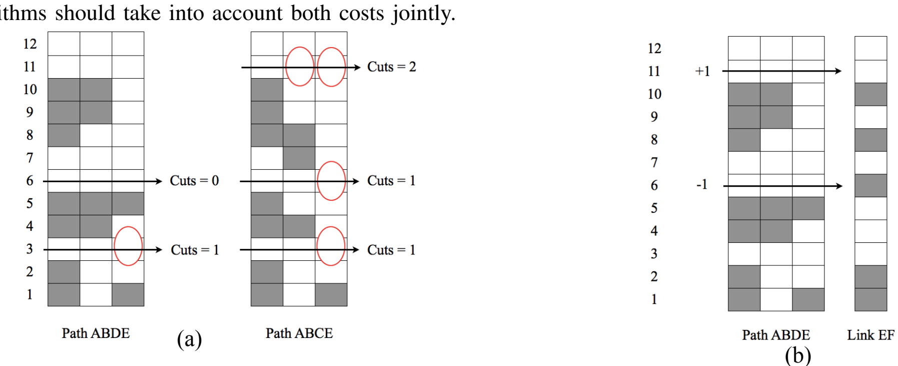


Fig. 3. a) FS "cuts"and b) "misalignment produced by slot assignment to a demand.

In [13] is defined a new parameter F_{cmt}, the fragmentation ratio which considers cuts, misalignments, and traffic as follows:

$$F_{\text{cmt}} = F_c + \frac{F_m}{S \times N} + H \times \frac{S}{C}, \tag{1}$$

in which:

- F_c = number of cuts;
- F_m = number of misalignment increases;
- S = number of slots requested by one connection;
- N = number of neighbor links for the candidate path;
- C = residual capacity of the candidate path;
- H = number of hops of the candidate path.

Among all solutions the minimum F_{cmt} indicates the least fragmentation solution when the traffic load is low and indicates the least congested solution when the traffic load is high.

III. Paths algebra framework

The algorithm proposed by [13] clearly separates the routing and slot assignment problems. Furthermore, the routing strategy is exclusively based number of hops and no Quality of Service (QoS) is taken into account. In fact, considering QoS is also insufficient as other optimization criteria are of interest: Quality of Transport (QoT), Quality of Network Economics (QoNE), Quality of Energy (QoEN).

It is necessary to conceive an heuristic or an algorithm to ensure the routing convergence for different types of QoX (X = S, T, NE, EN) metrics or QoX metrics composition, in which this problem could be addressed from an integrated and generic manner by means of a mathematical framework which allows validating the proposed solutions independently from network topology or implementation details.

Therefore, besides establishing a homogeneous mathematical basis, the concepts of paths algebra used in this work developed [14] and extended to solve the Virtual Network Embedding (VNE) problem [15] provide a guideline for developing a traffic engineering adaptive tool in which users can define their own path searching policy that can be closer to the existing traffic profile of their networks.

A. Paths Characterization

A network is represented by a directed graph $G = (V, A)$, where V is the set of vertices and A the set of arcs. Consider the simple path represented in Fig. 4.a. The set of vertices is given by $V = \{1, 2, 3, 4\}$ and the set of arcs is given by $A = \{a, b, c\}$. The source and destination nodes are $(s, d) = (1, 4)$. This path can be represented either as a succession of vertices $p_{1,4}$ or as a succession of arcs $p_{a,c}$.

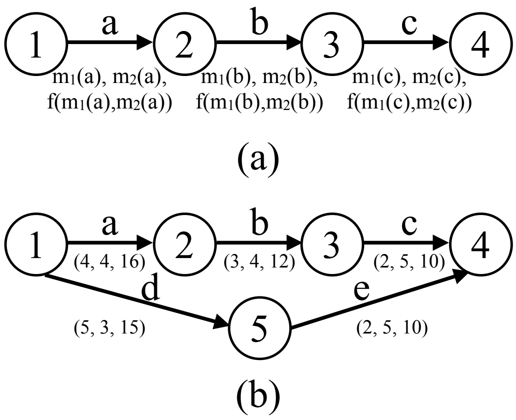


Fig. 4. (a) Example of a simple path (b) Example of two paths to be ordered

Each arc in this example is characterized by a triple $(m_1(x), m_2(x), f[m_1(x), m_2(x)])$, where: $m_1(x)$ and $m_2(x)$ are the values of metrics m_1 and m_2 on the arc $x \in A$; $f[m_1(x), m_2(x)]$ is a function of combination of metrics applied to $m_1(x)$ and $m_2(x)$.

In general, the paths algebra uses $\mathbf{M}$ as the set of m adopted routing metrics and $\mathbf{F}$ as the set of k metrics combination functions.

The set of combined-metrics of all edges is given by:

$$\overline{C}(p_{a,c}) = \begin{bmatrix} \overline{C}_a \\ \overline{C}_b \\ \overline{C}_c \end{bmatrix} = \begin{bmatrix} m_1(a) & m_2(a) & f\left[m_1(a), m_2(a)\right] \\ m_1(b) & m_2(b) & f\left[m_1(b), m_2(b)\right] \\ m_1(c) & m_2(c) & f\left[m_1(c), m_2(c)\right] \end{bmatrix}$$

A synthesis $\overline{S}[.]$ is a set of binary operations applied on the values of the links combined-metrics along a path to obtain a resulting value that characterizes this path as far as the constraint imposed by the combined-metric is concerned. So far, the syntheses are restricted to the following set: $\{\text{add}(), \text{mult}(), \text{max}(), \text{min}()\}$.

If the routing algorithm is mono-constraint, only one value is obtained as the synthesis result and it is called weight-word. If the routing algorithm is multi-constraint, with k constraints, then k values are obtained. In this example, $\overline{S}[.] = [S_1 S_2 S_3]^t$. The weight-word has as many letters as the path's number of arcs. The first letter corresponds to resulting value of the synthesis applied to the whole path; the second letter corresponds to resulting value of the synthesis applied to the subpath obtained by dropping out the last arc; the last letter corresponds to the resulting value of the synthesis applied to the subpath made of only the first arc. Any number of letters can be retained as the synthesis result and this is called an abbreviation: $\overline{b}_j\left(\overline{S}[.]\right)$ represents a j-letters abbreviation; $\overline{b}_\infty\left(\overline{S}[.]\right)$ represents no abbreviation, i. e., all letters are taken into account.

B. Paths Ordering

Consider the network represented in Fig. 4.b where two paths connect the source node 1 to the destination node 4. These paths are $\alpha = (1, 2, 3, 4) = (a, b, c)$ and $\beta = (1, 5, 4) = (d, e)$. Each paths' arc is characterized by a triple $(m_1(x), m_2(x), f\left[m_1(x), m_2(x)\right])$, where $f\left[m_1(x), m_2(x)\right] = m_1(x) \times m_2(x)$. The syntheses to be used in this example are given by $\overline{S}[.] = [\text{min}()\text{max}()\text{add}()]^t$.

TABLE II
SYNTHESIS RESULT OF THE NETWORK GIVEN IN FIG. 4.B

Path	S_1 min	S_2 max	S_3 add
α	2; 3; 4	5; 4; 4	38; 28; 16
β	2; 5	5; 3	25; 15

The result of the synthesis is shown in Table II. A path α is worse or less optimized than a path β, if $\overline{S}[\alpha] \preceq_{ML} \overline{S}[\beta]$, where $\preceq_{ML}$ stands for multidimensional lexical ordering. In the example $\preceq_{ML} = \{\geq, \leq, \geq\}$, that is translated by the following ordering relations:

- $S_1[\alpha] \preceq S_1[\beta] \Rightarrow S_1[\alpha] \geq S_1[\beta]$;
- $S_2[\alpha] \preceq S_2[\beta] \Rightarrow S_2[\alpha] \leq S_2[\beta]$;
- $S_3[\alpha] \preceq S_3[\beta] \Rightarrow S_3[\alpha] \geq S_3[\beta]$;

Different syntheses also have different priorities. In the example, S_1, S_2 and S_3 priorities go from the highest to the lowest.

Table III summarizes the results obtained for three different ordering criteria. It is important to realize that the syntheses letters are examined from the highest priority to the lowest priority synthesis. When the paths are considered equivalent, then we will examine either the next letter of the same synthesis or will move to the next synthesis. This is determined by the adopted abbreviation.

IV. FS-AWARE RSA PROBLEM MAPPING

As shown in Section III, the paths algebra enables to explore different optimization strategies by just changing the metrics $\mathbf{M}$ and the function of combined metrics $\mathbf{F}$.

Table IV shows how some different strategies can be modeled using the paths algebra. In this table, the optimization metrics are primarily used to order the enumerated paths. The physical constraints indicate if the achieved mapping is feasible or not. A feasible mapping requires positive physical constraints. A QoS threshold is a lower priority metrics and represents a bound on the value of the corresponding metrics.

Different strategies may be exploited to optimize business objectives and achieve different levels of QoS. The listed strategies also mix linear and non-linear metrics that treated simultaneously by the paths algebra.

The set of metrics shown in Table IV can be augmented to take into account the FS-aware RSA problem associated metrics.

Equation (1) is a combined-metrics that can be evaluated and ordered according to the syntheses and ordering relations indicated in Table V

TABLE III
PATHS ORDERING OF THE NETWORK GIVEN IN FIG. 4.B

Abbreviation $\overline{b}_j\ (\overline{S}[.])$	Result
$\overline{b}_1[S_1]\ \overline{b}_1[S_2]\ \overline{b}_1[S_3]$	$S_1 \Rightarrow \alpha \equiv \beta$
	$S_2 \Rightarrow \alpha \equiv \beta$
	$S_3 \Rightarrow \alpha \prec \beta$
$\overline{b}_\infty[S_1]\ \overline{b}_\infty[S_2]\ \overline{b}_\infty[S_3]$	$S_1 \Rightarrow$ 1st letters are equal
	$\Rightarrow \alpha \equiv \beta$
	$S_1 \Rightarrow$ 2nd letters $\Rightarrow$
	$3 < 5 \Rightarrow \beta \prec \alpha$
$\overline{b}_1[S_1]\ \overline{b}_\infty[S_2]\ \overline{b}_1[S_3]$	$S_1 \Rightarrow \alpha \equiv \beta$
	$S_2 \Rightarrow$ 1st letters are equal
	$\Rightarrow \alpha \equiv \beta$
	$S_2 \Rightarrow$ 2nd letters $\Rightarrow$
	$4 > 3 \Rightarrow \beta \prec \alpha$

TABLE IV
MODELING OF OPTIMIZATION STRATEGIES

Optimization criterion	**Optimization metrics**	**Physical constraints**	**QoS thresholds**	$\mathbf{M}$ $\mathbf{F}$
Minimize cost	Hops	CPU, BW	-	$\mathbf{M} = \{\text{Hops}, \text{BW}, \text{CPU}\}$ $\mathbf{F} = \mathbf{M}$
Minimize cost under a maximum allowable delay	Hops	CPU, BW	Delay	$\mathbf{M} = \{\text{Hops}, \text{Delay}, \text{BW}, \text{CPU}\}$ $\mathbf{F} = \mathbf{M}$
Maximize the spare CPU	CPU	CPU, BW	-	$\mathbf{M} = \{\text{CPU}, \text{BW}\}$ $\mathbf{F} = \mathbf{M}$
Maximize the spare BW	BW	CPU, BW	-	$\mathbf{M} = \{\text{BW}, \text{CPU}\}$ $\mathbf{F} = \mathbf{M}$
Maximize the spare physical resources	CPU, BW	CPU, BW	-	$\mathbf{M} = \{\text{BW}, \text{CPU}\}$ $\mathbf{F} = \{\text{CPU} + \text{BW}\}$
Minimize cost and maximize throughput, under a maximum allowable delay	Hops, PLR	CPU, BW	Delay	$\mathbf{M} = \{\text{Hops}, \text{PLR}, \text{Delay}, \text{BW}, \text{CPU}\}$ pTHRU = 1 - PLR $\mathbf{F} = \{\text{Hops}, \text{pTHRU}, \text{Delay}, \text{BW}, \text{CPU}\}$

TABLE V: FS-aware RSA problem associated metrics, syntheses and ordering relations

Metrics	**Synthesis**	**Ordering relation ($\preceq_{ML}$)**
F_c = number of cuts	add	$\geq$
F_m = number of misalignment increases	add	$\geq$
C = residual capacity of the candidate path	min	$\leq$
H = number of hops of the candidate path	add	$\geq$
Combined metrics F_{cmt} = the fragmentation ratio	add	$\geq$

In the evaluation of Equation (1), S = (number of slots requested by one connection) and N = (number of neighbor links for the candidate path) are constants for the specific demand and chosen path.

The FS-aware RSA problem for the the network represented in Figure 2 for the FSs occupancy given in Table I has been mapped into the paths algebra framework. For the sake of simplicity, it has been considered for routing purposes the number of hops as the highest priority metrics. Accordingly, the path ADE is selected. The example has been done for a 1 FS demand. In this case, $N = 5$, $H = 2$, $C = 7$, and $S = 1$. Table VI shows the obtained result.

TABLE VI: FS-aware RSA problem mapped into the paths algebra framework

Slot	$F_c =$	$F_m =$	$F_{\text{cmt}} =$
2	0	1	0.486
3	2	5	3.286
4	0	1	0.486
8	1	1	1.486
9	2	1	2.486
10	2	1	2.486
11	2	5	3.286
12	0	5	1.286

Applying the ordering relation ($\preceq_{ML}$=$\geq$) results that the best slots are slots 2 and 4.

It is important to notice that the routing objective criteria can be easily changed to take into account QoX parameters. The fragmentation ratio F_{cmt} = can be chosen to be the highest priority metrics and the QoX criteria can be used as thresholds to define candidate routes. The shortest path does not have to necessarily the best choice. In summary, the paths algebra framework provides flexibility to exploit different FS-aware RSA strategies.

V. Conclusions and Future Works

In this work we proposed the use of the paths algebra framework to solve the FS-aware RSA problem. The framework allows to exploit different optimization strategies taking into account QoX parameters. There is no restriction neither for the number nor for the type (linear / non-linear) of metrics. The proposal has been described by means of the example described in [13]. Other FS-aware RSA formulations can be easily mapped into the framework.

As future work we intend to apply the paths algebra framework to propose strategies for the SLA-aware RSA problem in EONs.

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